Dispersed Shock Waves in a Gas-Particle Mixture

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Nomenclature

a = sound speed, cm/s
C = specific heat of particles
C_D = drag coefficient
C_{pg}, C_{vg} = specific heats of gas at constant pressure and volume, respectively
k_g = thermal conductivity of gas
m = ratio of mass flow rates of particles and gas
M = Mach number
n = $m(C/C_{pg})$
N = Nusselt number
Pr = Prandtl number
Re = Reynolds number
r_p = particle radius, cm
t = temperature
T = dimensionless temperature, t/t_∞
v = velocity
V = dimensionless velocity, v/v_∞
x = dimensional distance
X = dimensionless distance, $\alpha\sqrt{T_1}x/M_\infty$
γ_g = ratio of specific heats, C_{pg}/C_{vg}
δ = specific heats ratio, C/C_{pg}
ϵ = volume of particles in a unit volume of mixture
μ_g = viscosity of gas, poise
ρ_g = density of gas, gm/cm^3

Subscripts

g = gas
p = particles
1 = upstream of shock wave
∞ = at downstream infinity

Introduction

FEW attempts have been made to analyze dispersed or weak shock waves in a gas-particle mixture. Kriebel,[1] Rudinger,[2,3] and, more recently, Hamed and Frohn[4] have studied the structure of fully dispersed shock waves in dusty gases. In all of these investigations, particle volumes have not been included. The assumption to neglect the particle volume is well justified. However, if either the mass fraction of the particles or the gas density is sufficiently high, the particle volume fraction may become significant. Its effect has been evaluated by Srivastava and Sharma[5] for discontinuous normal shock waves. In the present study, an attempt has been made to obtain the structure of dispersed shock waves by including the particle volume effect. The effect of the drag and heat-transfer coefficients on the structure of dispersed shock waves have also been ascertained.

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Mathematical Formulation

Following Kriebel,[1] the modified flow equations including the particle volume effect are given as follows:

$$(T_g-1)+(T_p-1)n+M_\infty^2\frac{(\gamma_g-1)}{2}[V_g^2-1+(V_p^2-1)m]$$
$$+\epsilon_\infty\frac{(\gamma_g-1)}{2}\left[\frac{T_gV_p}{V_g(V_p-\epsilon_\infty)}-\frac{1}{(1-\epsilon_\infty)}\right]=0 \tag{1}$$

$$(V_g-1)+(V_p-1)m$$
$$+\frac{1}{\gamma_gM_\infty^2}\left[\frac{T_gV_p}{V_g(V_p-\epsilon_\infty)}-\frac{1}{(1-\epsilon_\infty)}\right]=0 \tag{2}$$

$$\frac{\mathrm{d}V_p}{\mathrm{d}x}=-\frac{C_DRe}{24}\frac{\alpha\sqrt{T_1}}{M_\infty}\frac{(V_p-V_g)}{(V_p-\epsilon_\infty)} \tag{3}$$

where $\alpha=9\mu_g/2r_p^2\rho_pa_1$

$$V_p\frac{\mathrm{d}T_p}{\mathrm{d}x}=\frac{Nk}{2}\frac{\alpha\sqrt{T_1}}{M_\infty}(T_g-T_p) \tag{4}$$

where $k=2k_g/3C\mu_g$.

Setting Up Boundary Conditions

The method of Rudinger[2] is adopted for the analysis of dispersed shock waves. It is based on the linearization of the basic equations and yields the perturbations in terms of an initially chosen perturbation without further assumptions.

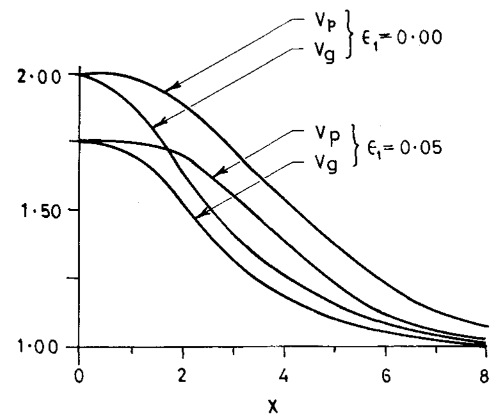

Fig. 1 Gas and particle velocities vs nondimensional distance X behind shock wave for $\epsilon_1=0$ and 0.05.

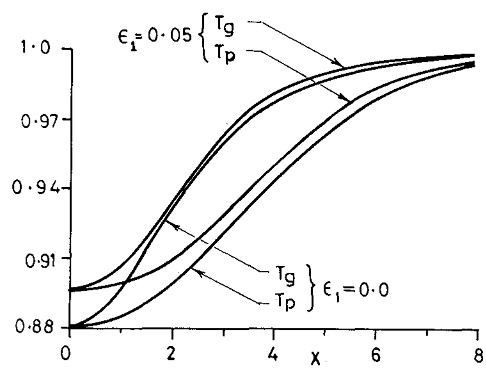

Fig. 2 Gas and particle temperatures vs nondimensional distance X behind shock wave for $\epsilon_1=0$ and 0.05.

In a linearized system of equations, the perturbations must vanish exponentially as x approaches $-\infty$. The flow variables in the range $-\infty \leq x \leq 0$ may thus be expressed in the form

$$V_g = V_1 - V_g' e^{\lambda x} \tag{5}$$

$$V_p = V_1 - V_p' e^{\lambda x} \tag{6}$$

$$T_g = T_1 + T_g' e^{\lambda x} \tag{7}$$

$$T_p = T_1 + T_p' e^{\lambda x} \tag{8}$$

where the primed quantities represent the small starting perturbations at $x=0$ and $\lambda>0$ is a constant to be determined.

Substituting Eqs. (5-8) into Eqs. (1-4), one obtains a set of linear (higher powers of the perturbation being neglected) homogeneous algebraic equations for which a nontrivial solution exists only if the determinant of the coefficients vanishes. The resulting quadratic equation for λ is

$$\begin{aligned}&\{(1-M_1^2)-\epsilon_1[(1-2M_1^2)]\}\left(\frac{\lambda}{\alpha}M_1\right)^2+\{1-(1+m)M_1^2\\&-\beta(\bar{M}_1^2-1-m)-\epsilon_1[1+(1+m)M_1^2\\&+\beta(2\bar{M}_1^2-1-m)]\}\left(\frac{\lambda}{\alpha}M_1\right)\\&-\beta(1+m)[(\bar{M}_1^2-1)-\epsilon_1(\bar{M}_1^2+1)]=0\end{aligned} \tag{9}$$

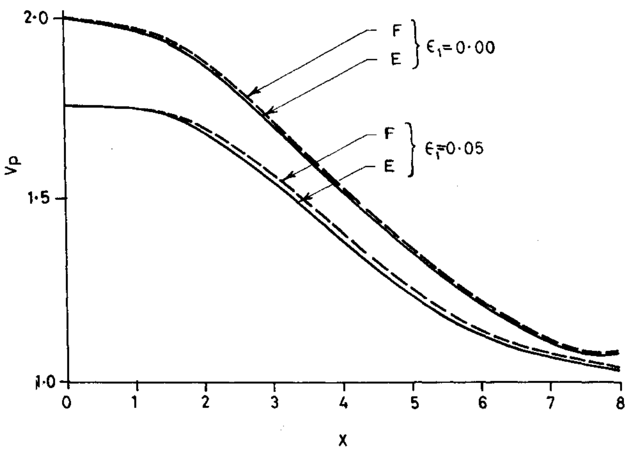

Fig. 3 Particle velocity vs nondimensional distance X behind shock wave for $C_D=24/Re$, $N=2+0.533\ Re^{0.5}$ (curves E) and for $C_D=24/Re$, $N=2$ (curves F), both for $\epsilon_1=0$ and 0.05.

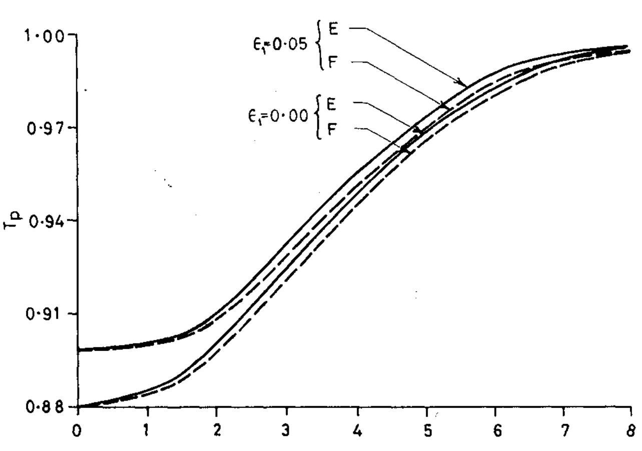

Fig. 4 Particle temperature vs nondimensional distance X behind shock wave for $C_D=24/Re$, $N=2+0.533Re^{0.5}$ (curves E) and for $C_D=24/Re$, $N=2$ (curves F), both for $\epsilon_1=0$ and 0.05.

where

$$\bar{M}_1^2=\frac{(m+1)(n\gamma_g+1)}{(n+1)}M_1^2$$

$$\epsilon_1=\frac{\epsilon_\infty}{V_1}, \qquad \beta=\frac{m(1+n)}{3n(1+m)}\frac{N}{Pr} \tag{10}$$

Once the value of λ has been obtained, the system of algebraic equations for the starting perturbation, i.e.,

$$V_p'=V_g'\left(1+\frac{\lambda}{\alpha}M_1\right)^{-1}+\epsilon_1 V_g'\left(\frac{\lambda}{\alpha}M_1\right)\left(1+\frac{\lambda}{\alpha}M_1\right)^{-2} \tag{11}$$

$$\begin{aligned}T_g'&=\gamma_g M_1 M_\infty\sqrt{T_1}\left[mV_p'-\left(\frac{1}{\gamma_g M_1^2}-1\right)V_g'\right]\\&-\epsilon_1\gamma_g M_1 M_\infty\sqrt{T_1}\left[\left(\frac{1}{\gamma_g M_1^2}+m\right)V_p'+V_g'\right]\end{aligned} \tag{12}$$

$$\begin{aligned}T_p'&=\frac{1}{n}[(\gamma_g-1)M_1M_\infty\sqrt{T_1}(V_g'+mV_p')-T_g']\\&-\frac{\epsilon_1}{n}\frac{(\gamma_g-1)}{\gamma_g}(T_g'+M_1^{-1}M_\infty\sqrt{T_1}V_g')\end{aligned} \tag{13}$$

are readily solved by taking V_g' as the arbitrarily selected value to insure that all of the perturbations remain small. Since $V_1 \geq V_g \geq 1$, a small fraction of (V_1-1) represents a suitable choice of V_g'. Boundary conditions at $x=0$ are obtained from Eqs. (5-8) by putting $x=0$. Using these boundary conditions, the solution for $x>0$ is obtained by solving numerically Eqs. (1-4). For $x<0$, the values are obtained from Eqs. (5-8).

Numerical Calculations and Discussion of Results

Numerical calculations have been performed to obtain the velocity and temperature distribution for the gas and particles in an air-silica mixture (Figs. 1 and 2) and to ascertain the effects of C_D and N on the particles in the mixture (Figs. 3 and 4) for $\epsilon_1=0.0$ and 0.05 plotted against a nondimensional distance X, assuming $Pr=2/3$, $\delta\approx 1$, $\gamma_g=1.4$, $m=1$, $n=1$, and $M_1=0.95$.

For $C_D=24/Re$ (Stokes' law) and $N=2$, the gas and particle velocity distribution is depicted in Fig. 1 and the temperature distribution in Fig. 2. The velocity of the particles in both cases remains higher than that of the gas, although their respective values show a significant decrease when the effect of particle volume is considered. Also, the equilibrium in particle volume case is attained earlier. Regarding the temperature distribution, the gas temperature remains hgher than the particle temperature in the two cases, but their respective values decrease when the particle volume is incorporated.

Shock relaxation zones for the particles have been computed for $C_D=24/Re$ and for $N=2+0.533Re^{0.5}$ (curves E) and $N=2$ (curves F) for both $\epsilon_1=0.0$ and 0.05. The graphs show that the change in Nusselt number has very little effect on the velocity distribution (Fig. 3), but has some effect on the temperature distribution (Fig. 4) in the two cases. Also, the velocity of the particle remains higher for $\epsilon_1=0.0$ than for $\epsilon_1=0.05$, while the particle temperature remains lower for $\epsilon_1=0.0$ than for $\epsilon_1=0.05$.

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Conservation Form of the Equations of Fluid Dynamics in General Nonsteady Coordinates

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Introduction

MANY of the differential equations that arise in the field of fluid dynamics may be stated in conservation-law form. A well-known example is, of course, the Navier-Stokes equations.

Recent interest in the generation of general body-oriented curvilinear coordinate systems for solving these equations has given rise to many forms of presentations of the conservation form of equations in curvilinear coordinates. Generally speaking, the conservation-law form of the equations seems definitely preferable, particularly when shocks or other discontinuities form part of the admissible solutions. In the numerical sense, it is also essential to formulate discretization schemes for the governing equations that satisfy conservation requirements.

Several previous works on the subject of deriving the conservation-law form of the Navier-Stokes equations in general nonsteady coordinate systems have been reported in the literature.[1-5] The purpose of this Note is to illustrate a mathematical methodology with which such forms of the equations may be derived in an easier and more general fashion.

For numerical purposes, the scalar form of these equations is eventually presented in Cartesian components. Satisfactory numerical results using the conservation-law form of the equations are impossible to obtain, unless the numerical scheme employed also respects geometric conservation in a discrete sense.

Conservation Form of Equations in Curvilinear Coordinates

The general conservation law for classical fields is stated in integral form[5] as

$$\int_V \frac{\partial A}{\partial t} \mathrm{d}V + \int_S f \cdot n \mathrm{d}S = \int_V C \mathrm{d}V \tag{1}$$

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or, in differential form, as

$$\frac{\partial A}{\partial t} + \mathrm{div} f = C \tag{2}$$

where n is the unit outward normal to the surface S which encloses the material volume $V(t)$. The quantities A, f, and C are tensors such that f is of one order higher than A or C, where C is the source term.

As stated in Ref. 5, if Eq. (2) is written (in curvilinear coordinates) in the strong conservation-law form, i.e., in a form in which undifferentiated terms do not appear, then the conservation law will be preserved.

Suppose that physical space $L^3(E_1^c, E_2^c, E_3^c)$[§] is spanned by the base vectors E_i^c $(i=1,2,3)$. L^3 may be extended to the four-dimensional linear space $L^4(E_1^c, E_2^c, E_3^c, E_4^c)$, where the vector E_4^c (denoting time t) is defined as satisfying $E_i^c \cdot E_4^c = \delta_{i4}$, δ being the usual Kronecker delta. We now set

$$B = f + AE_4^c = B_c^\alpha E_\alpha^c \tag{3}$$

$$\nabla = E_\alpha^c \frac{\partial}{\partial X^\alpha} \tag{4}$$

Then Eq. (2) may be written in the divergence form in L^4 space as

$$\nabla \cdot B = C \tag{5}$$

The general curvilinear coordinate system may be enunciated by the following mapping:

$$\xi^i = \xi^i(x^1, x^2, x^3, x^4) \qquad i=1,2,3$$

$$\xi^4 = x^4 \tag{6}$$

where $(x^1, x^2, x^3, x^4) = (x, y, z, t)$. In order to express Eq. (5) in curvilinear coordinates, use will be made of the following tensor formula for the divergence of a vector F:

$$\nabla \cdot F = \frac{1}{J} \frac{\partial}{\partial \xi^\alpha}(JF^\alpha) \tag{7}$$

where J is the Jacobian of the transformation from x^i to ξ^i.

By use of Eq. (7), Eq. (5), in a curvilinear coordinate system that includes time, may then be expressed as

$$\frac{\partial}{\partial \xi^\alpha}(JB^\alpha) = JC \tag{8}$$

The only difference in the treatment of the time and space variables is clear when it is observed that

$$\frac{\partial \xi^4}{\partial x^\beta} = \delta_{4\beta} \tag{9}$$

from Eq. (6). Thus,

$$B^4 = B_c^4 = A \tag{10}$$

$$B^i = B_c^\beta \frac{\partial \xi^i}{\partial x^\beta} = B_c^j \frac{\partial \xi^i}{\partial x^j} + B_c^4 \frac{\partial \xi^i}{\partial x^4}$$

$$= f_c^j \frac{\partial \xi^i}{\partial x^j} + A \frac{\partial \xi^i}{\partial t} = f^i + A\omega^i \tag{11}$$

[§]Except for x^i and ξ^j, the subscripts i, j, and k denote covariants, while the superscripts denote contravariant components in L^3. α and β have a similar interpretation in L^4. Repeated indices imply summation, while the subscript or superscript c denotes Cartesian components.